# A Method for Resolving Small Temporal Variations of Effective Elastic Properties

Götz H.R. Bokelmann[1]

## ABSTRACT

A method for studying temporal variations of wave propagation properties in the earth is specifically useful for studying crustal dynamics, but also for studying reservoir geophysics. Medium anisotropy causes three phases to propagate along almost the same path. This allows a determination of the relative difference of shear-wave velocities $\delta\hat{\beta}$ in a fashion, which is remarkably insensitive to potential errors. Even instrumental timing errors should not affect the results. If there are nearly identical doublet sources, which often occur naturally, one can determine differences of $\delta\hat{\beta}$ with extreme accuracy. This allows resolving small temporal variations of wave propagation properties, which cannot usually be detected within the larger measurement uncertainty of $\delta\hat{\beta}$. An application to data from a hydraulic fracturing experiment in a deep drilling borehole (KTB) showed that such temporal variations indeed exist, even at substantial depth levels in the crust. The relative difference of split shear-wave velocities decreased by about 2% during a 12-hour interval in the experiment. This can be explained only in terms of changing effective elastic properties of the medium. The method can be used with artificial and natural sources.

## INTRODUCTION

Variations of wave propagation properties with time have been searched for since the early days of modern seismology. In fact, there are a number of prominent rewards associated with this direction of research, which range from earthquake hazard reduction to production geophysics: Earthquake seismology may conceivably make use of the fact that the nature of wave propagation depends not only on the intrinsic structure of the medium, but also on external conditions such as the stress field (Nur and Simmons, 1969; Nur, 1971; Chesnokov and Zatsepin, 1991; Zatsepin and Crampin, 1995). Depending on changes in these external conditions, temporal variations of effective elastic properties (TVEEP) are expected. These variations are usually, but not always, associated with the space between individual grains in the typically composite medium. For simplicity, I will refer to the effective elastic properties simply as "elastic properties."

One focus of research in this field (e.g., Eisler, 1967,1969; Aki et al., 1970; DeFazio et al., 1973; Reasenberg and Aki, 1974; Poupinet et al., 1984; Aster et al., 1990; Haase et al., 1995; Nadeau et al., 1994) is to study changes before earthquakes to find systematic precursors of large earthquakes (Scholz et al., 1973). A number of such observations have been made using $v_P/v_S$-ratios (e.g., Semenov, 1969; Aggarwal et al., 1973; Lukk and Nersezov, 1978), coda-$Q$ (e.g., Aki 1985; Got et

[1] Institut für Geophysik, Ruhr-Universität Bochum, 44780 Bochum, Germany.

al., 1990; Got and Frechet, 1993; Beroza et al., 1995), and anisotropy (e.g., Peacock et al., 1988; Crampin et al., 1990; Booth et al., 1990; Crampin et al., 1991; Coutant, 1996). Many of these observations have been subsequently disputed (e.g., McEvilly and Johnson, 1974; Aster et al., 1990), so that it is not yet clear whether temporal variations can be observed in the context of crustal/earthquake seismology.

In production geophysics with oil and gas reservoirs the existence of TVEEP is less controversial: Effects induced in sedimentary layers by the extraction of fluids can be rather large, and numerous examples exist for changes of properties (e.g., Greaves and Fulp, 1987; Meadows and Winterstein, 1994), that are documented in amplitude variation with offset, $v_P/v_S$-ratio, and other quantities. In the field of reservoir geophysics, important applications of TVEEP-techniques are quite obvious.

While TVEEP are clearly present and measurable in production geophysics, unambiguous evidence for the effects occurring in a natural environment would be most useful. The existence of TVEEP in deeper regions of the crust was confirmed here using induced seismicity from a hydraulic fracturing experiment at 9-km depth in the KTB (Kontinentale TiefBohrung) borehole in Germany. In that fracturing experiment (Zoback and Harjes, 1997), about 400 induced events were recorded on a 76-station network at the surface and on a three-component borehole instrument in a second well at 4-km depth (minimum sampling rate, 1/2000 s). As we will see, there is clear evidence for temporal variations on time scales of hours that is documented in subtle changes of wave velocities. This method can be used with data from artificial and natural sources. Further evidence about temporal variations from it in both fields of earthquake and sedimentary seismology is therefore expected.

## METHOD

### Temporal variations

Elastic properties of the medium are described by the elasticity tensor $c_{ijkl}(\mathbf{x}, t)$. Testing for TVEEP with seismic waves requires repeated coverage of the same volume $\Gamma$ (see Figure 1), so that systematic changes of $c_{ijkl}(\mathbf{x})$ within the volume $\Gamma$ can be detected. If the waveforms are sufficiently broadband, the volume $\Gamma$ is given by the (first) "Fresnel-volume" (Snieder and Lomax, 1996), which comprises all possible scatterer positions giving rise to constructive interference with the direct wave (time delay $\Delta T < 1/4$ of dominant period).

To ensure that we are indeed measuring TVEEP, it is necessary to distinguish TVEEP effects from alternative effects (see Table 1). When dealing with natural sources (earthquakes), the locations of different events usually differ. Instrumental effects, such as, for example, an imprecise timing signal, could also cause apparent TVEEP. Essentially, it is necessary to distinguish between two classes of temporal variations—the *variations in the wider sense,* which include wandering of seismic sources, etc., and instrumental timing errors. On the other hand, *variations in the narrow sense* are associated with changes in elastic properties of the medium. The method proposed here does not insist on identical sources and an exactly known time signal. Instead the method separates the two classes of effects, so that TVEEP can be determined even if temporal variations in the wider sense are present.

### Relative difference of shear velocities

The determination of elastic constants is generally severely limited by resolution. Due to the large uncertainties of $c_{ijkl}(\mathbf{x})$, it is not advisable to study temporal variations by comparing $c_{ijkl}(\mathbf{x}, t_1)$ and $c_{ijkl}(\mathbf{x}, t_2)$at times $t_1$ and $t_2$, since uncertainties of $c_{ijkl}(\mathbf{x})$ are usually larger than the likely temporal variations. Instead, consider the extraction of information from waveform observations to see how errors and alternative effects influence the observables. Then, use a certain function of $c_{ijkl}(\mathbf{x}, t)$, for which the temporal variation can be studied with high accuracy.

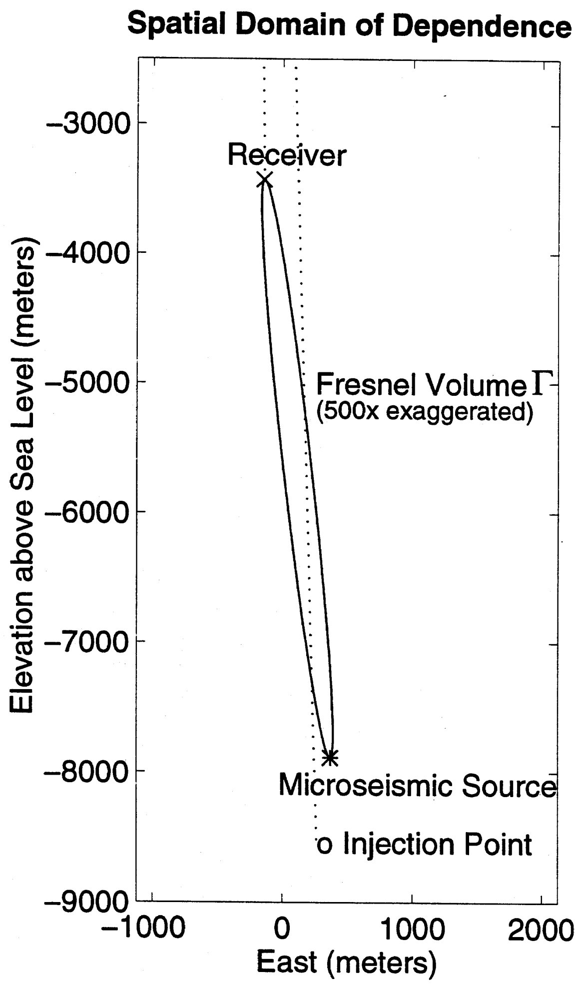


**FIG. 1. Observations of a seismic wave generated by a microseismic source depend on medium properties within the Fresnel-volume Γ (see text). Fluid injection at 9 km depth below the surface in the KTB borehole resulted in about 400 microseismic events. Many of these fall into distinct clusters of remarkably similar and nearly collocated events. These "doublets" allow coverage of the same volume Γ at different times, so that temporal variations of elastic properties $c_{ijkl}(x,t)$ in the volume Γ can be studied.**

**Table 1. Types of temporal variations.**

1. Temporal Variatons in the *Wider Sense*
   (Nuisance Effects)
   - Changes in Source Locations
   - Changes in Source Mechanisms
   - Instrumental Variations (Timing Errors and Clock Drift)

2. Temporal Variations in the *Narrow Sense*
   Temporal Variations of Effective Elastic Properties due to
   I) Change of the Medium
   - Fluid Content
   - Crack Geometry

   II) Changing Physical Conditions
   - Stress Field
   - Temperature

In plane-wave propagation in an anisotropic medium, shear waves are split into two orthogonally polarized waves. Primarily, we are interested in "shear-wave splitting" that can be described by the relative difference of shear-wave velocities $\delta\hat{\beta} = (\beta_1 - \beta_2)/\beta_1$. Along with the $P$-wave phase, there are, thus, three phases propagating along almost the same path (see Figure 2), so that we may consider two time differences

$$t_{S1} - t_P = \int \frac{1}{\beta_1} \frac{\eta - 1}{\eta} ds \approx \frac{D}{\beta_1} \frac{\eta - 1}{\eta} \tag{1}$$

and

$$t_{S2} - t_{S1} \approx \int \frac{\delta\hat{\beta}}{\beta_1} ds \approx \frac{D}{\beta_1} \delta\hat{\beta} \tag{2}$$

with the path length $D$ and $\eta = \alpha/\beta_1$. Equation (2) is correct to first order. Equating the distance $D$ in equations (1) and (2), we obtain

$$\delta\hat{\beta} = \frac{\beta_1 - \beta_2}{\beta_1} = \frac{t_{S2} - t_{S1}}{t_{S1} - t_P} \frac{\eta - 1}{\eta} . \tag{3}$$

Equation (3) has several interesting properties: Since $\delta\hat{\beta}$ depends only on time differences, it is not affected by the absolute time. Thus $\delta\hat{\beta}$ would not be influenced by errors in the time base of the recording instrument, which are not uncommon at the level of accuracy required to observe tempo-

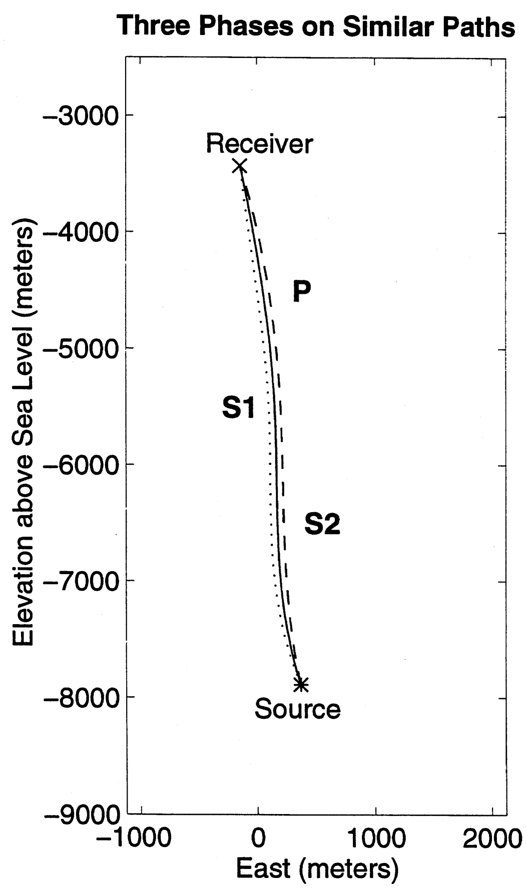


**FIG. 2. In a smoothly varying anisotropic medium there are three phases (*P*, *S*1, *S*2) that propagate approximately along the same path. The estimate of the relative difference of shear-wave velocities $\delta\hat{\beta} = (\beta_1 - \beta_2)/\beta_1$ is independent of timing errors (time offsets or clock drift) of the recording instrument and does not explicitly depend on the source location; see the discussion of equation (3).**

ral variations (see below). Since $\delta\hat{\beta}$ depends only on a quotient of time differences, it is also unaffected by a time drift, because that would introduce the same scaling constant in both the numerator and denominator, if the drift is constant over the short time interval between the arrival of *P*- and *S*-waves. Equation (3) does not explicitly depend on the distance; $\delta\hat{\beta}$ is the average over the Fresnel volume Γ. Thus, no accurate source location is needed; only sources that are sufficiently close to one another so that the waves generated by individual events are affected by approximately the same volume Γ. A simple procedure for solving this problem is treated in the section on doublet sources.

Taking $\eta = \sqrt{3}$, we can estimate $\delta\hat{\beta}$ from the three arrival times of the phases *P*, *S*1 and *S*2. Figure 3 shows a waveform example recorded on the KTB borehole instrument. The horizontal components are oriented 43 degrees (northeast) and 317 degrees (northwest), which approximately correspond to slow and fast directions of shear-wave propagation. There is clear shear-wave splitting with $\delta\hat{\beta} \approx 1\%$. Variations of this quantity with time are likely to be smaller than the estimated uncertainty of ± 0.3%, so that we wish to determine $\delta\hat{\beta}$ more accurately. Based on visual determination of shear-wave splitting, however, this uncertainty can not be improved substantially.

A different way of estimating $\delta\hat{\beta}$ is

$$\delta\hat{\beta} = \frac{(t_{S2} - t_{S1})^{ref} + \Delta(t_{S2} - t_{S1})}{(t_{S1} - t_P)^{ref} + \Delta(t_{S1} - t_P)} \frac{\eta - 1}{\eta} \tag{4}$$

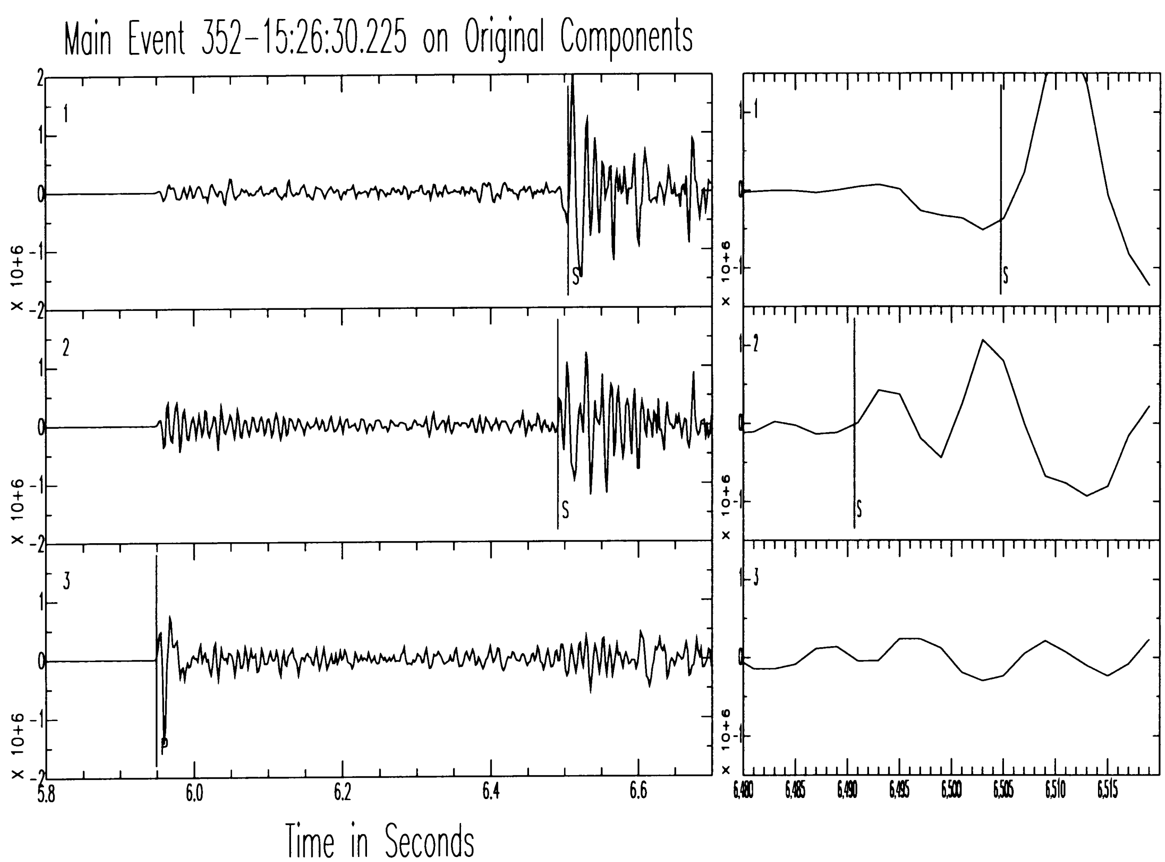


**FIG. 3. An example event from the KTB Fracturing Experiment (geometry of Figure 1) shows clear indication of shear-wave splitting with $\delta\hat{\beta}$ = 1 ± 0.3 %. The typical uncertainty of $\delta\hat{\beta}$ generally exceeds the size of temporal variations. On the other hand, $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$ can be determined with an uncertainty which is smaller by an order of magnitude (see text).**

which still has all the desirable properties of equation (3). The time differences $(t_{S2} - t_{S1})^{ref}$ and $(t_{S1} - t_P)^{ref}$ must be determined visually for a reference event. Hence, they are invariably associated with the large uncertainty as in the example of Figure 3. On the other hand, $\Delta(t_{S2} - t_{S1})$ and $\Delta(t_{S1} - t_P)$ can be determined by matching waveforms for two events: For example,

$$\Delta(t_{S2} - t_{S1}) = (t_{S2} - t_{S1}) - (t_{S2} - t_{S1})^{ref} =$$

$$(t_{S2} - t_{S2}^{ref}) - (t_{S1} - t_{S1}^{ref}) = \Delta t_{S2} - \Delta t_{S1} \quad (5).$$

$\Delta t_{S1}$ is the time delay between the fast shear-wave phases of the two events, which can be determined very accurately by a matched filter technique. If the waveforms of the two events are similar, then the uncertainty associated with the time delays $\Delta(t_{S2} - t_{S1})$ and $\Delta(t_{S1} - t_P)$ can reach an order of $10^{-1}$ ms or smaller.

To study temporal variation, it is sufficient to determine differences $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$. If we use the procedure as described, we can estimate $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$ accurately, despite the larger reference event errors. This property is demonstrated in the Appendix.

**Doublet sources**

An important component of a method for studying temporal variations of elastic properties is the source of seismic energy, which may be of artificial or natural origin. Artificial sources have been used in several experiments (e.g., Karageorgi et al., 1992). Even in that seemingly ideal circumstance, special considerations are required, e.g., from varying near-surface conditions. Here, I use naturally occurring sources. The method can, however, be similarly applied to artificial source data.

When using natural sources, it is important to make sure that the wave paths from any considered event cover essentially the same region Γ (Figure 1). That seems to require accurate source localization, which may be difficult to perform with sufficient accuracy. In fact, a simple waveform similarity test renders this unnecessary: Figure 4 shows an illustration of the kinematic interference pattern that results from the complex interaction of the wavefield with the medium heterogeneities. The two events, A and B, should give rise to quite different interference patterns of these waves due to the large separation between A and B. If A and B are close together, the patterns become more similar. This is the basis of the λ/4-criterion (Geller and Mueller, 1980), which states that, for good waveform correlation to exist, the two events should not be separated by more than λ/4. By testing for a high degree of waveform agreement, one can thus identify pairs of nearly collocated events.

Application of the method described in the last section also allows testing the λ/4-criterion to some degree: Figure 7 will show that the source-receiver distances of the events used here vary by less than λ/4, suggesting that the criterion is satisfied.

Figure 5 shows time windows for a group of four events (cluster 9 in Figure 5) containing both fast and slow shear-wave phases (normalized). The degree of waveform similarity is indeed very high, producing maximum values of the crosscorrelations of typically more than 0.95.

By testing waveform similarity of pairs of events and performing a cluster analysis, the set of 400 events is subdivided into groups (clusters) of events (Schulte-Theis, 1995). This provides the opportunity to study the temporal variation of elastic properties with high accuracy over the associated time intervals. Figure 6 shows the temporal distribution of six of the clusters with the highest waveform similarity.

## TEMPORAL VARIATION

By examining cluster 1 in Figure 6 the results can be studied in detail (Figure 7). The 13 events in cluster 1 cover an interval of about 12 hours. Over that interval $\delta\hat{\beta}$ decreases by about 2%. Cluster 10 in Figure 6 shows the same behavior. This variation can be explained only by temporal variations in the narrow sense.

The upper panel in Figure 7 shows estimates of $t_{S1} - t_P$ for the 13 events. Estimates from waveform matching, using the denominator of equation (4), are compared with visual estimates. Note that the scatter is smaller by at least an order of magnitude. This suggests, that (1) the waveform matching technique is far more accurate than simple visual arrival time estimation and (2) source-receiver distances vary by less than 6 m. This shows that, with regard to the source-receiver distance, the λ/4-criterion is, in fact, satisfied. Thus, the volume sensed by the individual events is indeed the same.

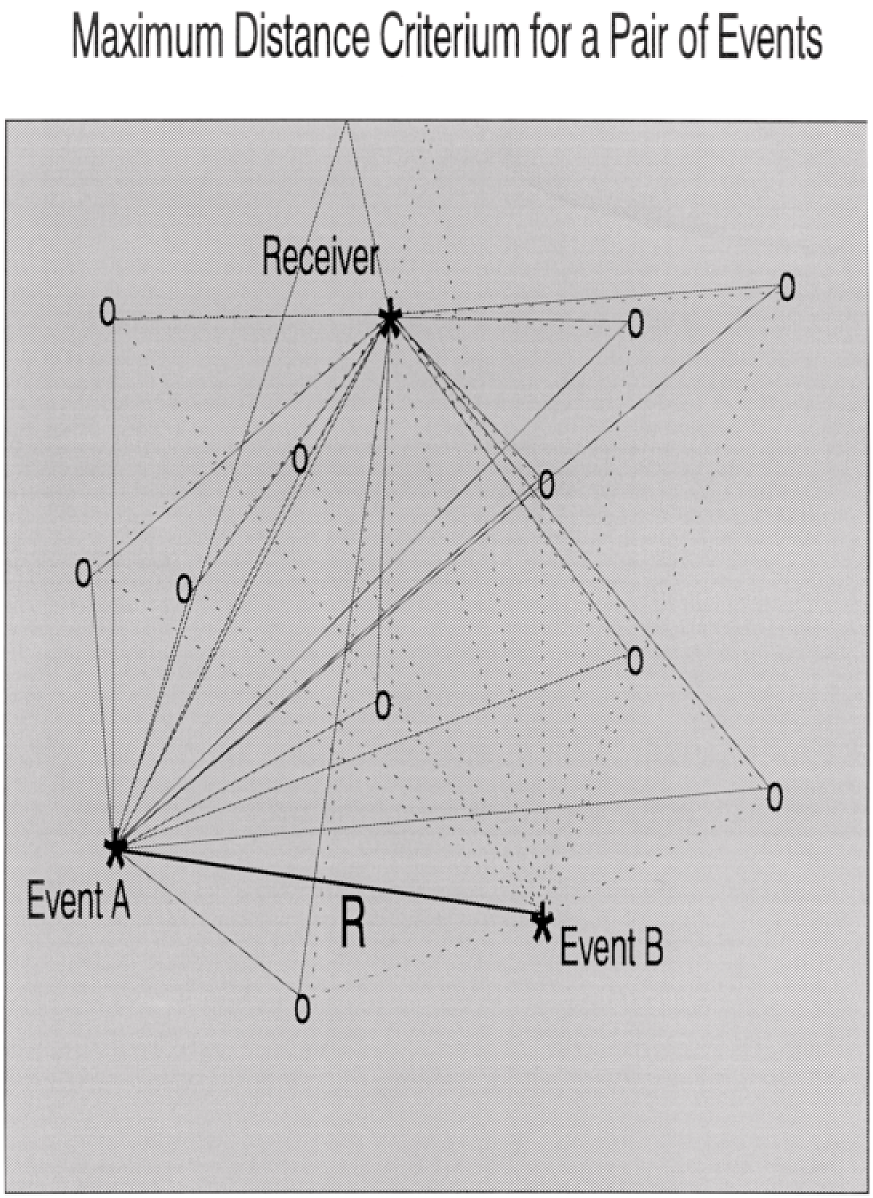


**FIG. 4. Illustration of the λ/4 maximum-distance criterion for similar events (doublets). Two seismograms of high similarity are associated with events, which occur close to each other (see text) with a distance of at most λ/4. Only then, the kinematic and dynamic pattern of scattered waves (reflections, conversions etc) can be matched.**

## CONCLUSIONS

An explanation for the variations of $\delta\hat{\beta}$ in Figure 7 must be sought in a temporal variation of the effective elastic properties of the medium. Other effects given in Table 1 (temporal variations in the wider sense) can be excluded, which is an important property of this method. In this fashion, important evidence is given demonstrating the existence of temporal variations of elastic properties in crustal seismology. The nature of the changes we observe can be associated with stress release during seismicity, the effect of injected fluid, and/or a tidal effect. These effects will be discussed elsewhere.

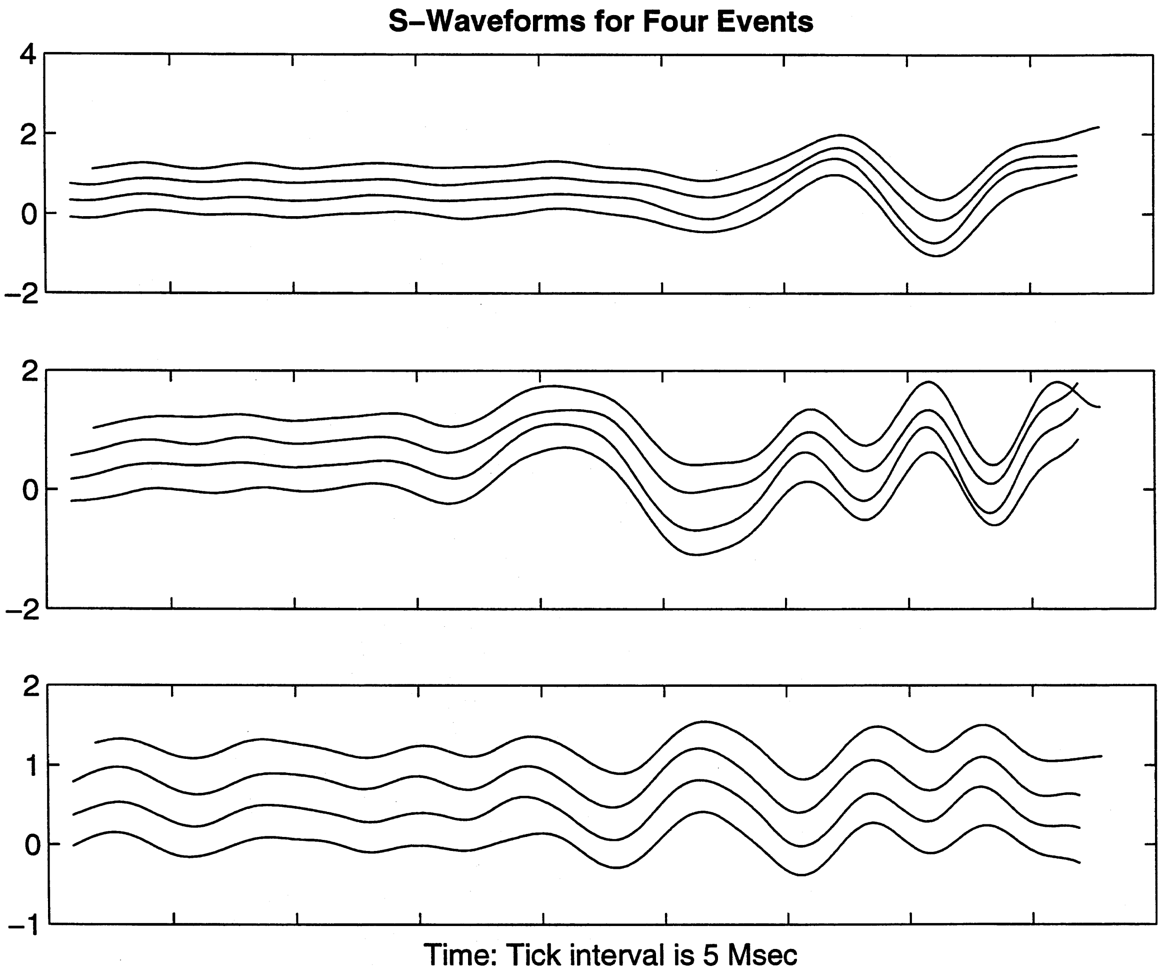


**FIG. 5. Example of high waveform similarity: *S*-waves for the four events in cluster 9 (see Figure 6).**

While the method was presented with an application to naturally occurring microseismicity data, it may be applied to artificial source data as well. Also in that field, the stabilizing properties of this technique may be very useful. Another application is to laboratory data, if high accuracy of variations of elastic properties is required.

If true source locations and the precise timing are known, one might simply determine $\alpha$, $\beta_1$, and $\beta_2$, which would give more constraints on the elastic constants and their temporal variation. However, these criteria are usually not met. This approach was, instead, to estimate temporal variations in a fashion that is not affected by these factors, whereas using the changes in wave velocities does not provide the accuracy required for observing subtle variations of $Q$ (attenuation).

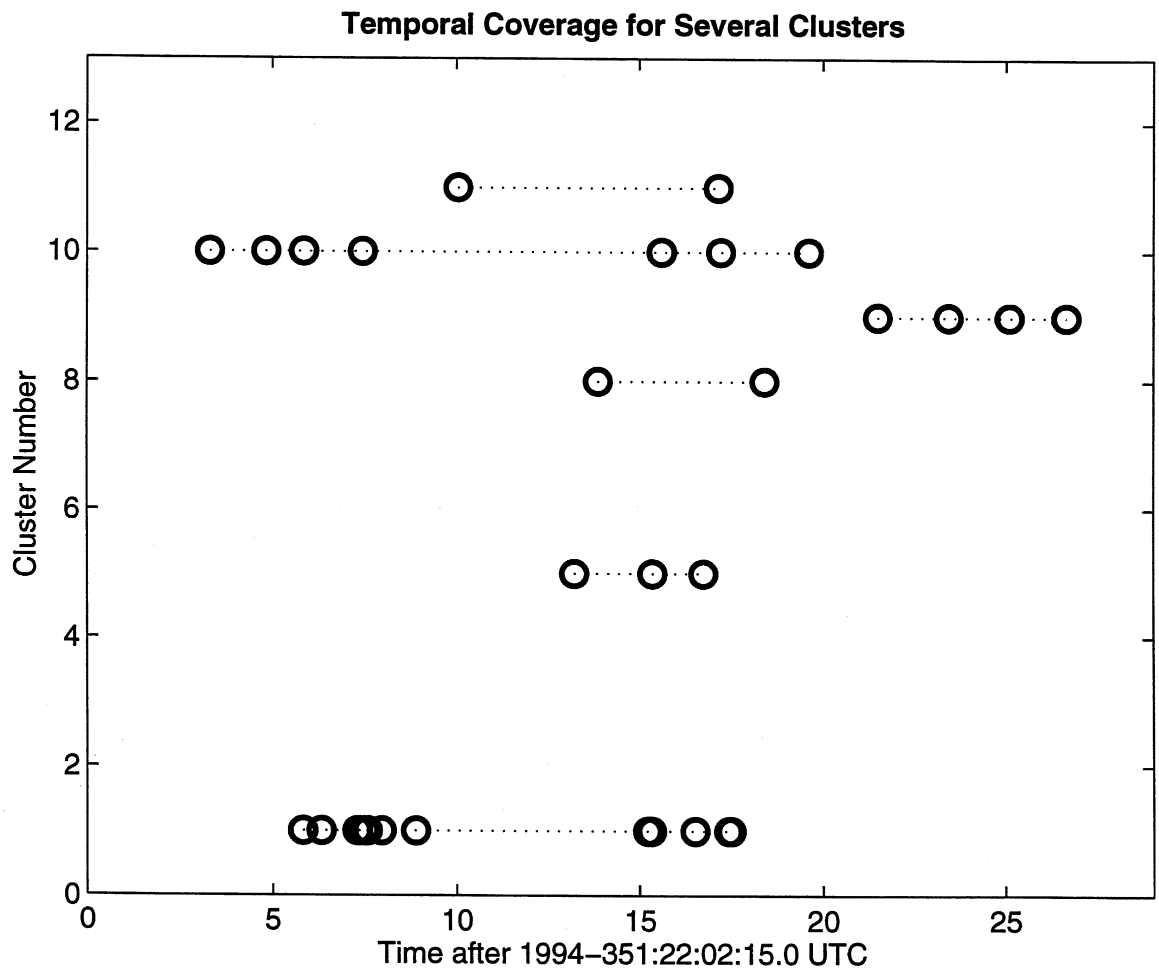


**FIG. 6. Temporal distribution of several clusters of events for the early part of the experiment. Each of the time windows shown by a dotted line allows a high-resolution study of temporal variations of elastic properties.**

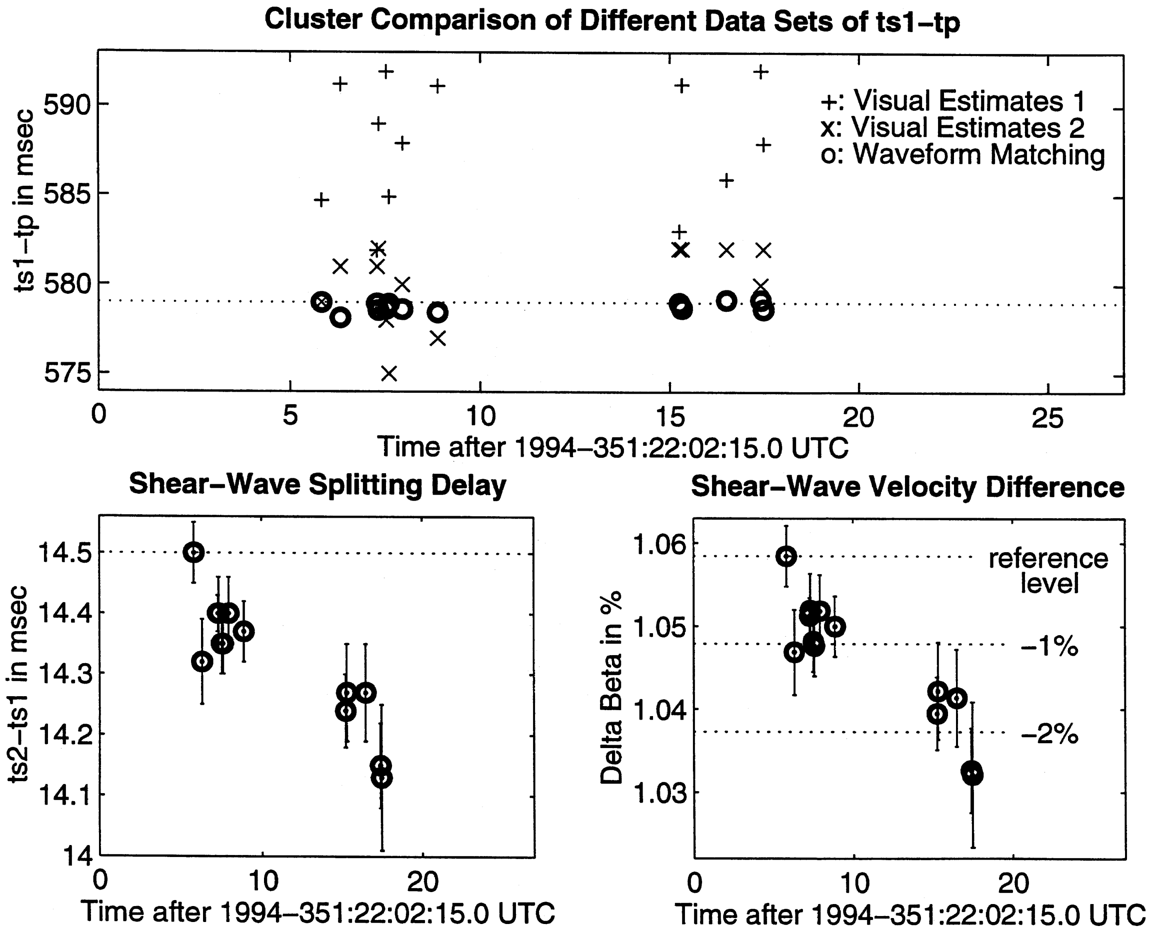


**FIG. 7. Results for the 13 events of cluster 1: The top and the lower left show intermediate steps for obtaining the change of $\delta\hat{\beta}$ shown on the lower right. The time differences $t_{S1} - t_p$ in the top panel show that the waveform matching method gives delays that are more accurate than manual estimates by at least an order of magnitude. While these delays are nearly constant for all events, the delay $t_{S2} - t_{S1}$ varies (lower left panel). This is reflected in the changing relative shear-wave velocity difference, which decreases by about 2% within 12 hours. This variation can only be explained by temporal variation of elastic properties.**

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## APPENDIX:

## ERROR ANALYSIS OF THE TEMPORAL VARIATION

Using (4) we can write $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$ in a form

$$\delta\hat{\beta} - \delta\hat{\beta}^{ref} = A\Big[\frac{a_1 + b_1}{a_2 + b_2} - \frac{a_1}{a_2}\Big] \tag{Al}$$

with $A = (\eta - 1)/\eta$, $a_1 = (t_{S2} - t_{S1})^{ref}$, $b_1 = \Delta(t_{S2} - t_{S1})$, $a_2 = (t_{S1} - t_P)^{ref}$ and $b_2 = \Delta(t_{S1} - t_P)$. $a_1$, $b_1$, $a_2$ and $b_2$ are measured quantities, which come with uncertainties. We write these as true value and error $a_1 = a_1^t + \delta_{a_1}$, $b_1 = b_1^t + \delta_{b_1}$, $a_2 = a_2^t + \delta_{a_2}$ and $b_2 = b_2^t + \delta_{b_2}$. If $a_2^t \gg \delta_{a_2} + b_2$ and $a_2^t \gg \delta_{a_2}$, we can expand the denominators and obtain

$$\delta\hat{\beta} - \delta\hat{\beta}^{ref} = \frac{A}{a_2^t}\Big[b_1(1 - \delta_{a_2} - b_2^t - \delta_{b_2} + \ldots)$$

$$+a_1(1 - \delta_{a_2} - b_2^t - \delta_{b_2} + \ldots) - a_1(1 - \delta_{a_2} + \ldots)\Big]$$

$$\approx \frac{A}{a_2^t}\Big[b_1(1 - \delta_{a_2} - b_2^t - \delta_{b_2}) - a_1^t(b_2^t + \delta_{b_2}) - \delta_{a_1}(b_2^t + \delta_{b_2})\Big] \tag{A2}$$

The estimates of $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$ are affected by (larger) errors $\delta_{a_1}$ and $\delta_{a_2}$ from the visual determination of time delays and (smaller) errors $\delta_{b_1}$ and $\delta_{b_1}$ from the waveform matching. In the leading orders the effect of $\delta_{a_1}$ and $\delta_{a_2}$ on $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$ are through $b_1^t\delta_{a_2}$ and $b_2^t\delta_{a_1}$, while the leading order of error from waveform matching errors is through $a_1^t\delta_{b_2}$.

It is easy to see that $b_1^t\delta_{a_2}$ and $b_2^t\delta_{a_1}$ are much smaller than $a_1^t\delta_{b_2}$: The size of $a_1^t$, $a_2^t$, $b_1^t$ and $b_2^t$ depend on the source-receiver distance *D*. The size of the errors $\delta_{a_1}$ and $\delta_{a_2}$ from visual arrival time picks depend primarily on characteristics of the waveform (spectral content etc.) while the size of $\delta_{b_1}$ and $\delta_{b_2}$ depend on waveform similarity between the two events. For the example of Figure 3 and other events in Bokelmann (1996a) we have effects of orders $[a_1^t] = 10^2\ ms$, $[b_1^t] = 10^{-1}\ ms$, $[\delta_{a1}] = 10^0\ ms$, $[\delta_{a_2}] = 10^0\ ms$ and $[\delta_{b_2}] = 10^{-1}\ ms$. Thus, we see that the contributions of errors $\delta_{a_1}$ and $\delta_{a_2}$ are formally two orders of magnitude smaller than those of the waveform matching errors. Determining $\delta\hat{\beta} - \delta\hat{\beta}^{ref}$ therefore allows an accurate determination of temporal variations (or stability) of elastic properties despite an inevitable large uncertainty in visual time determination.