

# Multicomponent noisy signal adaptive instantaneous frequency estimation using components time support information

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**Abstract:** This study proposes an adaptive method for components instantaneous frequency (IF) estimation of noisy non-stationary multicomponent signals, combined with the components time-support estimation method based on the short-time Rényi entropy (STRE). Components localisation and separation are done using a double-direction component tracking and extraction method presented here, while the IF estimation is done using the adaptive algorithms based on the intersection of confidence intervals (ICI) rule and the relative intersection of confidence intervals (RICI) rule. The results obtained using the ICI and RICI rules are compared for various window types, signal-to-noise ratios and time–frequency distributions, both with and without using the information on components time support. Most of the errors in IF estimation using the ICI and RICI-based methods are caused by imprecise components time-support estimation. The proposed methods combined with the STRE have achieved a significant accuracy improvement in terms of the mean absolute error and the mean squared error, reducing them by up to 73 and 93%, respectively. The method has been applied to real-life signals and proven to be an efficient tool for IF estimation of noisy non-stationary multicomponent signals.

## 1 Introduction

Instantaneous frequency (IF) is one of the basic signal parameters that provides important information about the time-varying spectral changes in non-stationary signals. The concept of IF finds its usage in various technical fields and applications, such as seismic, radar, sonar, communications and biomedical applications [1–11]. Different approaches for IF estimation have been proposed, starting with those efficient only for monocomponent signals. The IF estimation of a monocomponent signal becomes a more challenging task when the signal is corrupted by noise [3].

However, one of the most demanding IF estimation problems is finding the IF of each component in a multicomponent noisy non-stationary frequency modulated (FM) signal. Different techniques have been proposed, including the adaptive method based on the intersection of confidence intervals (ICI) rule and its modification referred to as the relative intersection of confidence intervals (RICI) rule [12]. The ICI and RICI rules were used for varying data-driven window width selection and were shown to outperform fixed size window IF estimation methods [13–15].

This paper presents improved IF estimation methods based on both the ICI and the RICI rules combined with the modified blind source separation (BSS) method (used for components localisation and extraction) [12], and the components time-support estimation method based on the

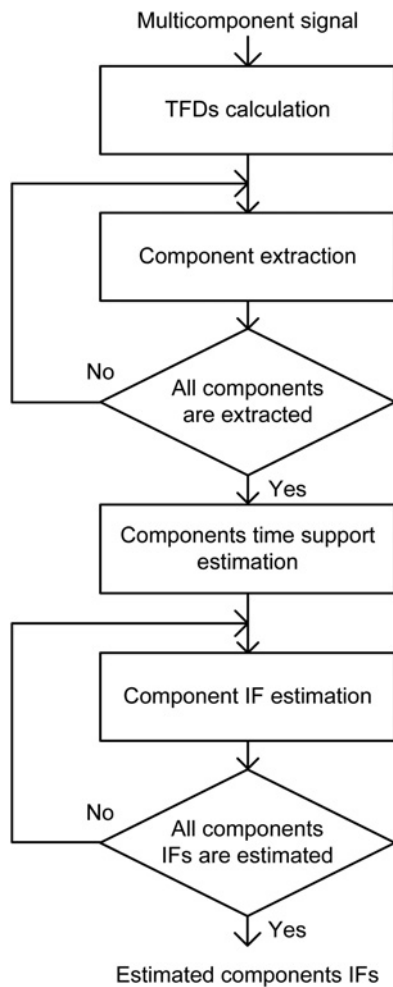
short-time Rényi entropy (STRE) [16]. The simplified flowchart of the method presented here is shown in Fig. 1. The paper is organised as follows. First, a signal component extraction method is presented in Section 2. Adaptive methods for multicomponent IF estimation based on the ICI and the RICI rules are described in Section 3. Components time-support estimation, based on the STRE, is discussed in Section 4. Finally, the results are presented and discussed in Section 5, followed by a conclusion in Section 6.

## 2 Signal components extraction

The signal components localisation and extraction in this paper are done using the modification of the algorithm in [17] (detailed in [12]). The algorithm extracts components one by one, until the remaining energy of the time-frequency distribution (TFD) becomes sufficiently small [18]. The algorithm consists of three steps, as follows:

*First step:* A reduced interference distribution (RID),  $\rho(t, f)$ , is calculated.

*Second step:* Components extraction is done based on the peaks in the signal RID, starting with the highest peak at  $(t_0, f_0)$  in the time–frequency domain which is then reduced to zero along with the frequency range neighbouring it, the size of which is  $F$ . Two TFD subregions are defined in the vicinity of  $(t_0, f_0)$  as  $f \in [f_0 - F/2, f_0 + F/2]$  (where  $t \in [t_0 - 1, t_0]$  and  $t \in$



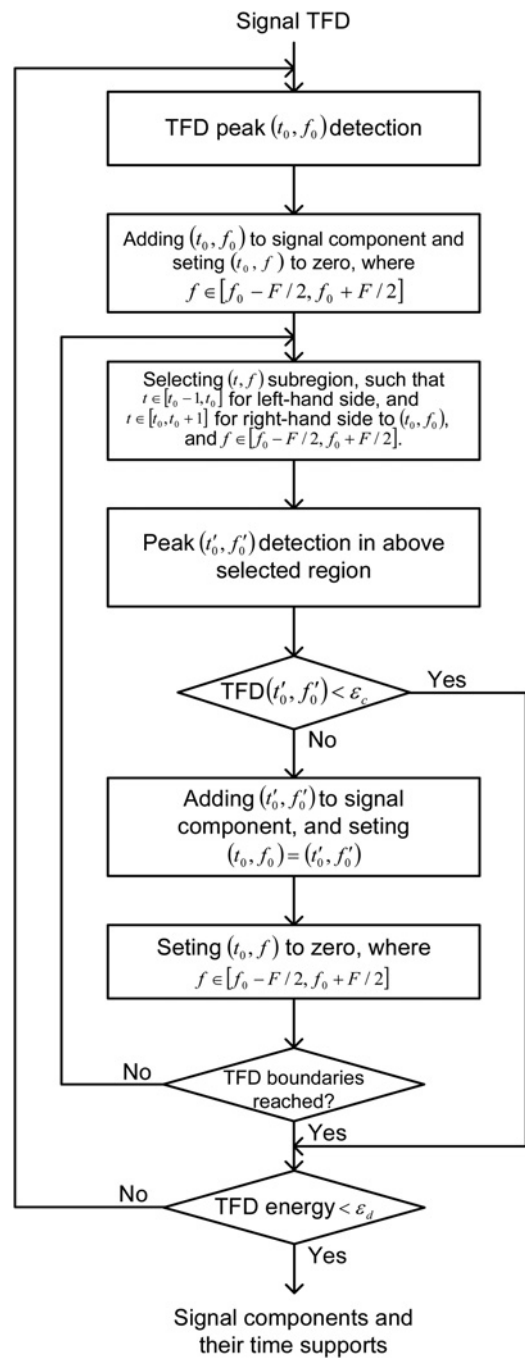
**Fig. 1** Simplified flowchart of the proposed IF estimation algorithm

$[t_0, t_0 + 1]$  for the first and the second subregions, respectively). For each subregion,  $(t'_0, f'_0)$  is calculated as the maximum of that subregion. The same extraction procedure is repeated until the local maxima values reach the threshold  $\epsilon_c$ , or the TFD limits are reached.

*Third step:* The preceding step of the method extracts a single signal component. So, the procedure is repeated for all components left in the TFD until the remaining TFD energy reaches the threshold value  $\epsilon_d$ .

The parameter  $F$  selection is both data and TFD dependent. Its choice, however, does not affect the IF estimation accuracy. Indeed, it mainly affects the component extraction quality due to the fact that the proposed IF estimation method is based on the component maxima locations and not on the component concentration itself. When selecting overly large  $F$  values, the extraction of parts belonging to other components may occur. Thus, the parameter  $F$  should be selected as small as possible, but still large enough to extract a whole component (not just a part of it). In the case of multicomponent signals, the distance between the components in the signal TFD must also be taken into consideration. The closer the components are, the smaller the value of  $F$  should be. Time–frequency distributions with high resolution (and hence high components concentration) allow smaller  $F$  values selection.

Being a double-direction component tracking method (unlike the method proposed in [17]), this method extracts



**Fig. 2** Flowchart of the adaptive signal components extraction procedure

whole components (unlike the one in [17] which extracts component segments that need to be combined into signal components by additional classification procedure). A flowchart of the adaptive components localisation and extraction procedure used in this paper is shown in Fig. 2.

Having all signal components extracted from its TFD, the IFs can be estimated using the adaptive methods described in the following section.

### 3 Adaptive multicomponent IF estimation

Once the components are localised and extracted from the signal TFD, the IF of each component can be obtained from the maxima in the TFD [3]. However, the IF estimated in this way exhibit estimation bias (caused by IF higher order

derivatives or noise) and estimation variance, which are shown to be the TFD analysis window length dependant; bias increases as the window length increases, unlike the estimation variance which decreases [14, 19, 20]. Hence, the estimation error reduction can be achieved by a proper window length selection that yields estimation bias to variance trade-off [14].

### 3.1 ICI-based multicomponent IF estimation algorithm

The ICI rule is an efficient method for the appropriate window width calculation as it does not require the bias estimation [13, 14]. Let us consider a model of signal in additive noise, defined as

$$y(n) = x(n) + \epsilon(n) \quad (1)$$

where

$$x(n) = \sum_{m=1}^M z_m(n) = \sum_{m=1}^M a_m(t) e^{j\phi_m(n)} \quad (2)$$

where  $\epsilon(n)$  is additive white Gaussian noise (real and imaginary zero-mean parts of variance  $\sigma_\epsilon^2/2$ ),  $a_m(t)$  and  $\phi_m(n)$  are the  $m$ th component instantaneous amplitude and instantaneous phase, respectively, and  $M$  is the number of components [3].

As shown in [12–14],  $\omega_m(n)$  belongs to the confidence interval  $D_m(n, l) = [L_m(n, l), U_m(n, l)]$  with probability  $P(\kappa)$ , where the upper  $U_m(n, l)$  and the lower  $L_m(n, l)$  limits of  $D_m(n, l)$  are calculated as

$$U_m(n, l) = [\hat{\omega}_m(n, h) + 2\kappa\sigma_m(h)] \quad (3)$$

$$L_m(n, l) = [\hat{\omega}_m(n, h) - 2\kappa\sigma_m(h)] \quad (4)$$

and  $h$  is taken from the set of increasing window widths  $H = \{h_1|h_1 < h_2 < \dots < h_J\}$ . The adaptive ICI-based IF estimation method calculates a sequence of TFDs for each of the window widths from  $H$  (chosen as in [12, 15]) followed by the components localisation and extraction, resulting in the  $m$ th component TFDs (TFD $_m(n, k, h)$ ) for each  $h$  from  $H$  and the set of  $J$  IFs estimates for each of the signal component. Then, the confidence intervals  $D_m(n, l)$  are calculated for each time instant and each window width  $h$ . As shown in [12–14], the ICI-rule-based IF estimation method tracks the intersection of  $D_m(n, l)$  and  $D_m(n, l-1)$ , giving the proper window width as the largest  $h$  which satisfies the following condition [12–14]

$$D_m(n, l-1) \cap D_m(n, l) \neq 0 \quad (5)$$

shown to provide the estimation error reduction and the estimation bias to variance tradeoff [12, 13].

### 3.2 RICI-based multicomponent IF estimation algorithm

As shown in [12, 15, 21], the ICI-rule-based window width selection can be improved by tracking the amount of overlap between the consecutive confidence intervals

$C_m(n, l)$  defined as

$$C_m(n, l) = |D_m(n, l) \cap D_m(n, l-1)| \quad (6)$$

where  $C_m(n, l)$  is normalised by the size of the current confidence interval

$$O_m(n, l) = \frac{C_m(n, l)}{|D_m(n, l)|} \quad (7)$$

As shown in [12, 15, 21], the  $O_m(n, l)$  value belongs to the interval  $[0, 1]$ , hence allowing to introduce the threshold value  $O_c$  as an additional criterion for the window width selection

$$O_m(n, l) \geq O_c \quad (8)$$

Using this additional criterion for calculating the TFD window width for each time instant results in a significant estimation error reduction [12, 15].

## 4 Components time-support estimation

The error in multicomponent signal IF estimation using the ICI-based and the RICI-based method is primarily caused by inaccuracy in components time-support estimation (see Fig. 3). In order to reduce the estimation error, one should use additional information on the components time support, which can be obtained from the STRE of the signal TFD [16]. The STRE-based method introduces short-time

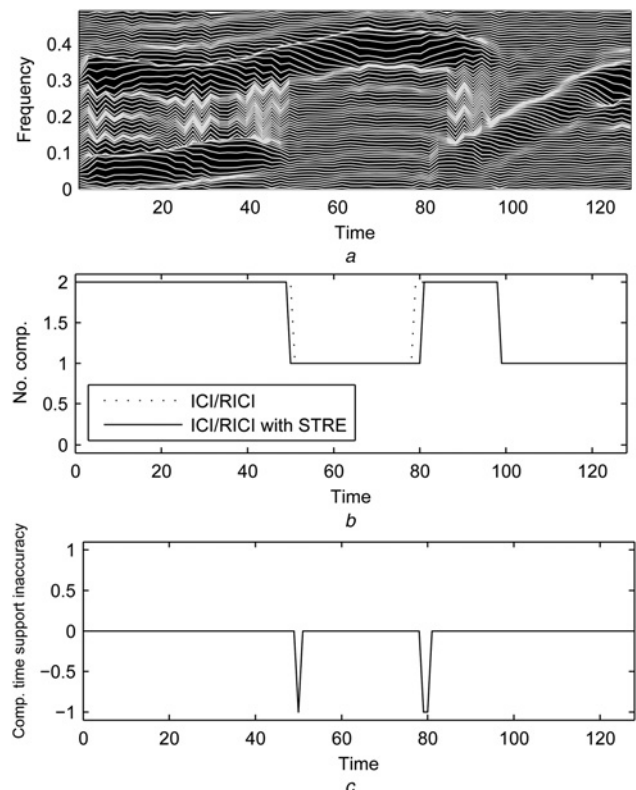


Fig. 3 Components time support

a Signal TFD  
 b Components time support obtained by the ICI and RICI-based methods with and without the STRE method  
 c Inaccuracy (difference) in components time-support estimation using the ICI and RICI-based methods when used without the STRE method

intervals in calculating the Rényi entropy, defined as

$$H_\alpha(\rho(t, f)) := \frac{1}{1 - \alpha} \log_2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho^\alpha(t, f) dt df \quad (9)$$

where the parameter  $\alpha$  is an odd integer that ensures a reduction in the oscillatory structure of interferences. The method calculates number of components over time as [16]

$$n(p) = 2^{H_\alpha(\rho_p(t, f)) - H_\alpha(\rho_p^{\text{ref}}(t, f))} \quad (10)$$

where  $\rho^{\text{ref}}(t, f)$  is the TFD of the reference signal, and  $p$  is the observed time instant.

This allows us to obtain the information about the number of signal components present at each time instant and compare it with the same information obtained using the ICI-based and the RICl-based methods (see Figs. 3b and c). The time supports obtained by the ICI and the RICl-based methods are corrected using the information provided by

the STRE, hence resulting in the IF estimation error reduction, as illustrated by examples in the following section.

## 5 Results and discussion

Let us consider a three-component signal  $x_1(n) = z_1(n) + z_2(n) + z_3(n)$ , where  $z_m(n) = A_m \exp(j\phi_m(n)) (A_m = 1)$ , as in [12], and a real-life signal of a dolphin sound. The achieved estimation error reduction in terms of the mean absolute error (MAE) and the mean squared error (MSE) using the information on the components time support is compared to the one obtained using the original ICI-based and RICl-based methods for various window types and different noise levels (defined as  $20 \log(A/\sigma_\epsilon)$ ). The component IF estimation MAE and MSE values are normalised by the component length.

The 128 samples signal  $x_1(n)$  contains two sinusoidal FM components and one linear FM component with different time supports (which partially overlap), defined by their IF laws as:  $\omega_1(n) = 0.35 + 0.05 \cos(2\pi(n - N_1/2)/N_1 - \pi/2)$ ,

**Table 1** IF estimation MAE comparison obtained using the RID for the methods based on the ICI and the RICl rule for the signal  $x_1(n)$  ( $\kappa = 1.75$ ,  $O_c = 0.97$ ,  $\epsilon_c = 0.1$ ,  $\epsilon_d = 0.01$ , rectangular time smoothing window of size  $(N/4) + 1$ , adaptive rectangular frequency smoothing window)

	20 log(A/σ <sub>ε</sub> )			Norm. MAE			
	ICI [× 10 <sup>-6</sup> ]	ICI + STRE [× 10 <sup>-6</sup> ]	Imp. ICI STRE [%]	RICl [× 10 <sup>-6</sup> ]	RICl + STRE [× 10 <sup>-6</sup> ]	Imp. RICl STRE [%]	Total Imp. [%]
<i>Component 1</i>							
5	118.05	115.61	2.06	92.11	89.67	2.64	24.04
10	90.99	88.66	2.57	75.11	72.78	3.11	20.02
15	78.38	76.04	2.98	75.19	72.85	3.11	7.05
20	73.18	70.84	3.19	72.43	70.09	3.22	4.21
<i>Component 2</i>							
5	131.06	118.07	9.91	123.07	110.09	10.55	16.00
10	99.55	73.97	25.69	96.15	70.43	26.75	29.25
15	92.48	79.49	14.04	91.11	78.12	14.25	15.52
20	81.47	68.76	15.61	81.47	68.76	15.61	15.61
<i>Component 3</i>							
5	122.04	109.74	10.08	110.42	98.12	11.14	19.60
10	114.73	87.65	23.61	102.95	75.63	26.54	34.08
15	83.49	71.19	14.73	80.54	68.24	15.27	18.27
20	78.50	66.20	15.67	78.16	65.86	15.74	16.11

**Table 2** IF estimation MSE comparison obtained using the RID for the methods based on the ICI and the RICl rule for the signal  $x_1(n)$  ( $\kappa = 1.75$ ,  $O_c = 0.97$ ,  $\epsilon_c = 0.1$ ,  $\epsilon_d = 0.01$ , rectangular time smoothing window of size  $(N/4) + 1$ , adaptive rectangular frequency smoothing window)

	20 log(A/σ <sub>ε</sub> )			Norm. MSE			
	ICI [× 10 <sup>-6</sup> ]	ICI + STRE [× 10 <sup>-6</sup> ]	Imp. ICI STRE [%]	RICl [× 10 <sup>-6</sup> ]	RICl + STRE [× 10 <sup>-6</sup> ]	Imp. RICl STRE [%]	Total Imp. [%]
<i>Component 1</i>							
5	215.91	215.89	0.01	211.65	211.62	0.01	1.99
10	136.55	136.52	0.02	133.87	133.85	0.02	1.98
15	135.23	135.20	0.02	134.93	134.91	0.02	0.23
20	129.76	129.74	0.02	129.78	129.75	0.02	0.01
<i>Component 2</i>							
5	39.51	39.38	0.33	35.70	35.57	0.36	9.98
10	27.22	15.83	41.84	25.77	14.32	44.44	47.41
15	26.71	26.58	0.49	26.56	26.43	0.49	1.06
20	13.79	13.66	0.92	13.78	13.66	0.92	0.97
<i>Component 3</i>							
5	72.20	72.08	0.17	70.59	70.46	0.17	2.41
10	74.08	56.28	24.02	72.95	55.13	24.42	25.58
15	58.61	58.49	0.21	58.28	58.16	0.21	0.77
20	53.12	53.00	0.23	53.11	52.98	0.23	0.26

**Table 3** IF estimation MAE comparison obtained using the MBD for the methods based on the ICI and the RICl rule for the signal  $x_1(n)$  ( $\beta=0.1$ ,  $\kappa=1.75$ ,  $O_c=0.97$ ,  $\epsilon_c=0.1$ ,  $\epsilon_d=0.01$ , adaptive rectangular time and frequency smoothing windows)

20 log ( $A/\sigma_\epsilon$ )	Norm. MAE						
	ICI [ $\times 10^{-6}$ ]	ICI + STRE [ $\times 10^{-6}$ ]	Imp. ICI STRE [%]	RICl [ $\times 10^{-6}$ ]	RICl + STRE [ $\times 10^{-6}$ ]	Imp. RICl STRE [%]	Total Imp. [%]
<i>Component 1</i>							
5	256.96	163.16	36.50	242.87	148.72	38.77	42.12
10	219.87	146.40	33.42	215.18	141.47	34.25	35.66
15	211.71	137.74	34.94	211.07	137.08	35.06	35.25
20	209.30	136.37	34.85	209.07	136.13	34.89	34.96
<i>Component 2</i>							
5	151.75	66.95	55.88	143.52	57.86	59.69	61.87
10	119.03	43.92	63.11	117.71	42.49	63.91	64.31
15	106.36	31.76	70.14	106.30	31.71	70.17	70.19
20	101.49	35.45	65.07	101.63	35.61	64.96	64.91
<i>Component 3</i>							
5	140.81	90.37	35.82	125.77	74.38	40.86	47.18
10	116.73	79.33	32.04	113.06	75.50	33.22	35.32
15	104.90	76.41	27.16	104.56	76.06	27.26	27.50
20	101.82	71.57	29.70	102.16	71.93	29.59	29.35

**Table 4** IF estimation MSE comparison obtained using the MBD for the methods based on the ICI and the RICl rule for the signal  $x_1(n)$  ( $\beta=0.1$ ,  $\kappa=1.75$ ,  $O_c=0.97$ ,  $\epsilon_c=0.1$ ,  $\epsilon_d=0.01$ , adaptive rectangular time and frequency smoothing windows)

20 log ( $A/\sigma_\epsilon$ )	Norm. MSE						
	ICI [ $\times 10^{-6}$ ]	ICI + STRE [ $\times 10^{-6}$ ]	Imp. ICI STRE [%]	RICl [ $\times 10^{-6}$ ]	RICl + STRE [ $\times 10^{-6}$ ]	Imp. RICl STRE [%]	Total Imp. [%]
<i>Component 1</i>							
5	704.08	403.01	42.76	701.01	401.40	42.74	42.99
10	640.15	403.50	36.97	640.87	403.71	37.01	36.94
15	638.52	400.29	37.31	638.53	400.30	37.31	37.31
20	630.89	399.66	36.65	630.89	399.65	36.65	36.65
<i>Component 2</i>							
5	106.86	17.56	83.56	104.95	15.46	85.27	85.53
10	85.94	7.62	91.13	85.78	7.44	91.32	91.34
15	67.24	4.62	93.13	67.25	4.63	93.12	93.12
20	56.09	6.69	88.07	56.10	6.70	88.06	88.06
<i>Component 3</i>							
5	61.34	32.34	47.27	57.90	29.42	49.20	52.04
10	63.13	38.16	39.55	62.19	37.19	40.21	41.10
15	51.78	40.58	21.63	51.77	40.57	21.63	21.65
20	51.52	38.25	25.76	51.54	38.27	25.74	25.72

**Table 5** IF estimation MAE comparison obtained using the RID for the methods based on the ICI and the RICl rule for the signal  $x_1(n)$  ( $20 \log(A/\sigma_\epsilon)=10$ ,  $\kappa=1.75$ ,  $O_c=0.97$ ,  $\epsilon_c=0.1$ ,  $\epsilon_d=0.01$ , time smoothing window of size  $(N/4)+1$ , adaptive frequency smoothing window)

Window type	Norm. MAE						
	ICI [ $\times 10^{-6}$ ]	ICI + STRE [ $\times 10^{-6}$ ]	Imp. ICI STRE [%]	RICl [ $\times 10^{-6}$ ]	RICl + STRE [ $\times 10^{-6}$ ]	Imp. RICl STRE [%]	Total Imp. [%]
<i>Component 1</i>							
Rect	90.99	88.66	2.57	75.11	72.78	3.11	20.02
Hamming	86.55	84.21	2.70	75.38	73.05	3.10	15.59
Hanning	93.66	91.32	2.49	86.62	84.28	2.70	10.01
Triang	101.72	99.38	2.30	94.75	92.42	2.46	9.14
Gauss	102.38	82.72	19.21	109.31	89.79	17.85	12.30
<i>Component 2</i>							
Rect	99.55	73.97	25.69	96.15	70.43	26.75	29.25
Hamming	98.25	85.26	13.22	91.33	78.34	14.22	20.26
Hanning	98.09	85.11	13.24	73.26	60.27	17.73	38.56
Triang	83.13	70.14	15.62	80.94	67.95	16.05	18.26
Gauss	112.95	99.97	11.50	99.51	86.52	13.05	23.40
<i>Component 3</i>							
Rect	114.73	87.65	23.61	102.95	75.63	26.54	34.08
Hamming	107.79	95.49	11.41	89.63	77.33	13.72	28.26
Hanning	140.75	92.56	34.24	133.11	83.36	37.38	40.78
Triang	125.38	84.26	32.80	116.24	73.65	36.64	41.26
Gauss	144.25	97.06	32.71	159.20	111.38	30.04	22.79

**Table 6** IF estimation MSE comparison obtained using the RID for the methods based on the ICI and the RICl rule for the signal  $x_1(n)$  ( $20 \log(A/\sigma_a) = 10$ ,  $\kappa = 1.75$ ,  $O_c = 0.97$ ,  $\epsilon_c = 0.1$ ,  $\epsilon_d = 0.01$  time smoothing window of size  $(N/4) + 1$ , adaptive frequency smoothing window)

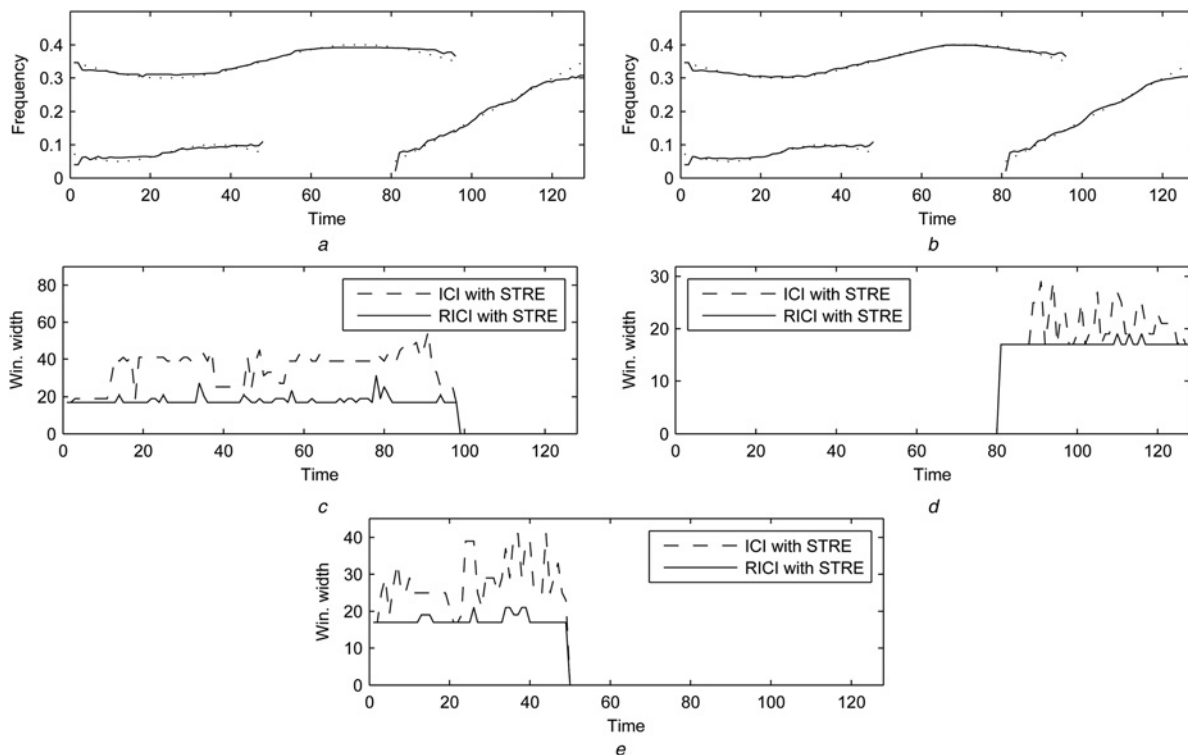
Window type	Norm. MSE						
	ICI [ $\times 10^{-6}$ ]	ICI + STRE [ $\times 10^{-6}$ ]	Imp. ICI STRE [%]	RICl [ $\times 10^{-6}$ ]	RICl + STRE [ $\times 10^{-6}$ ]	Imp. RICl STRE [%]	Total Imp. [%]
<i>Component 1</i>							
Rect	136.55	136.52	0.02	133.87	133.85	0.02	1.98
Hamming	129.90	129.88	0.02	126.47	126.44	0.02	2.66
Hanning	134.05	134.03	0.02	133.75	133.73	0.02	0.24
Triang	151.34	151.32	0.02	149.69	149.66	0.02	1.11
Gauss	158.05	115.91	26.66	160.06	118.32	26.08	25.14
<i>Component 2</i>							
Rect	27.22	15.83	41.84	25.77	14.32	44.44	47.41
Hamming	25.22	25.09	0.51	23.66	23.53	0.55	6.71
Hanning	19.05	18.92	0.68	12.13	12.00	1.07	37.03
Triang	22.94	22.81	0.57	22.07	21.94	0.59	4.32
Gauss	39.70	39.57	0.33	34.42	34.29	0.38	13.62
<i>Component 3</i>							
Rect	74.08	56.28	24.02	72.95	55.13	24.42	25.58
Hamming	60.59	60.47	0.20	58.89	58.76	0.21	3.01
Hanning	157.88	63.67	59.67	171.78	70.72	58.83	55.21
Triang	89.87	55.72	38.01	96.79	60.44	37.56	32.75
Gauss	160.92	71.38	55.64	171.63	77.70	54.73	51.71

$\omega_2(n) = 0.05 + 0.3(n - 1)/(N_2 - 1)$  and  $\omega_3(n) = 0.075 + 0.025\cos(2\pi(n - N_3/2)/N_3 - \pi/2)$ . The components time durations are  $N_1 = 96$ ,  $N_2 = 48$  and  $N_3 = 48$ .

The modified B-distribution (MBD) [20] and the RID from [22] are used with varying frequency smoothing window lengths, chosen from the set  $H$  which contains 25 increasing window lengths, with the number of frequency bins  $N_f = 4N$ ,

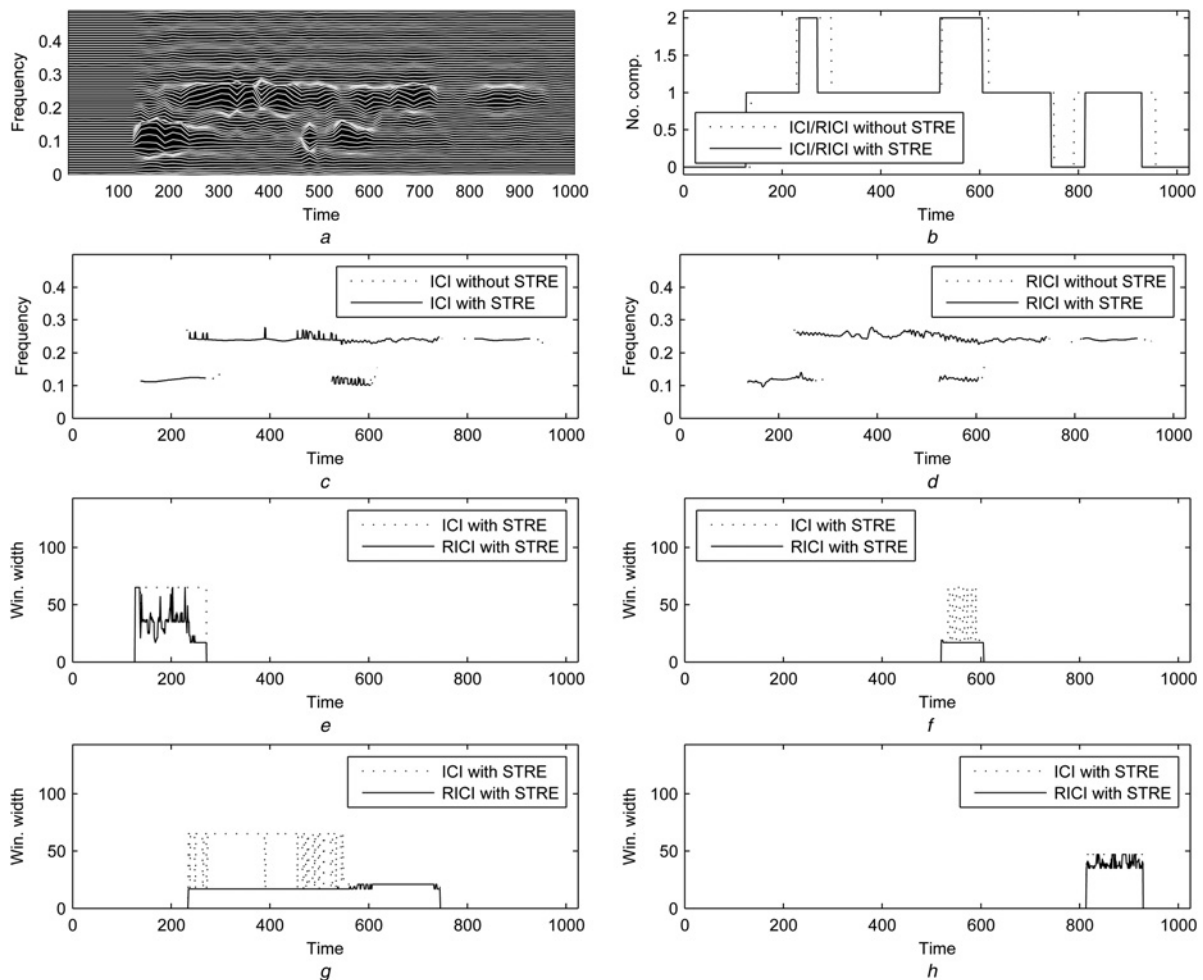
and the time smoothing window length of  $(N/4) + 1$ . For the component separation and extraction procedure  $F = N_f/4$ ,  $\epsilon_c = 0.1$  and  $\epsilon_d = 0.01$ . The parameter  $\kappa$  value used in both IF estimation methods, based on the ICI and the RICl rule, is set to be  $\kappa = 1.75$ , while  $O_c = 0.97$ .

Tables 1–4 show the IF estimation MAE and MSE for the ICI and the RICl-based method, with and without



**Fig. 4** IFs and adaptive window widths estimated using the ICI and RICl-based methods for signal  $x_1(t)$

- a Components IFs estimated using the ICI-based method combined with the time-support information (solid) and signal IFs (dotted)
- b Components IFs estimated using the RICl-based method combined with the time-support information (solid) and signal IFs (dotted)
- c Adaptive window width obtained using the ICI-based and the RICl-based method combined with the time support information for the first component
- d Adaptive window width obtained using the ICI-based and the RICl-based method combined with the time support information for the second component
- e Adaptive window width obtained using the ICI-based and the RICl-based method combined with the time support information for the third component



**Fig. 5** IFs and adaptive window widths estimated using the ICI and RIC-based methods for real-life signal (dolphin sound)

*a* Dolphin sound signal TFD

*b* Components time support obtained by the ICI and RIC-based methods with and without using the time-support information

*c* Components IFs estimated using the ICI-based method with (solid) and without (dotted) using the time-support information

*d* Components IFs estimated using the RIC-based method with (solid) and without (dotted) using the time-support information

*e* Adaptive window width obtained using the ICI-based (dotted) and the RIC-based (solid) method combined with the STRE method for the first component

*f* Adaptive window width obtained using the ICI-based (dotted) and the RIC-based (solid) method combined with the STRE method for the second component

*g* Adaptive window width obtained using the ICI-based (dotted) and the RIC-based (solid) method combined with the STRE method for the third component

*h* Adaptive window width obtained using the ICI-based (dotted) and the RIC-based (solid) method combined with the STRE method for the fourth component

information on the components time support for the MBD and the RID with the rectangular time and frequency smoothing windows for different noise levels  $20 \log(A/\sigma_\epsilon) = [5, 10, 15, 20]$ . The ICI-based and RIC-based methods proposed in this paper have improved using the components time-support estimation algorithm; they have led to significantly lower MAE and MSE estimation errors, reducing it by up to 70 and 93%, respectively.

Tables 5 and 6 give the MAE and MSE results obtained using the ICI-based method and its modification RIC-based method on the RID for  $20 \log(A/\sigma_\epsilon) = 10$  and different window types (rectangular, Hamming, Hanning, triangular and Gaussian), with and without using the information on the components time support. As it can be seen, the MAE reduction improvement is by up to 41% and the MSE is reduced by up to 55%.

The IFs estimated using the ICI and RIC-based methods and the components time support estimated using the STRE are shown in Figs. 4*a* and *b*, respectively. The adaptive window width obtained using the ICI and RIC-based methods with and without the STRE for all signal components can be seen in Figs. 4*c-e*.

Fig. 5 shows the IF estimation results for a real-life signal (dolphin sound). The signal TFD is shown in Fig. 5*a*, while the IFs estimated using the ICI and RIC-based methods both with and without the STRE are given in Figs. 5*b* and *c*. The components time support for each component estimated using the STRE for the two methods are shown in Figs. 5*d-h*. The most of the IF estimation error using the original ICI-based and RIC-based methods is caused by the inaccuracy in a proper component time support estimation. This has been improved by combining those methods with the components time-support estimation method based on the STRE, hence yielding a significant estimation error reduction (MAE is reduced by up to 70%, and MSE by up to 93%).

## 6 Conclusion

This paper presents an adaptive method for the IF estimation of a multicomponent noisy non-stationary signals, enhanced by the components time-support estimation method based on the STRE. It improves the performance of earlier IF

estimation method by incorporating time-support information. The Rényi entropy is used to exactly count the number of signal components at each time instant. Therefore, using Rényi entropy just after extraction of signal components overcomes the inaccuracy in the estimated time support of the extracted signal components in [12]. This problem of the BSS algorithm may be caused by the dependence on a predetermined fixed threshold, and so, this method overcomes the sensitivity of the overall method on the predetermined threshold. Localisation and extraction of components from a noisy signal mixture are performed using the double-direction component tracking method. The IF estimation is done using the adaptive methods based on intersection of confidence intervals rule upgraded by the components time-support information. Significant IF estimation quality improvements (reducing IF estimation mean absolute error by up to 70%, and mean squared error by up to 93%) has been achieved, as illustrated on both synthetic and real-life multicomponent signals.

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